Journal of Applied Mechanics and Technical Physics, Vol. 43, No. 3, pp. 457–466, 2002

MODEL OF A PLASTICALLY COMPRESSIBLE MATERIAL AND ITS APPLICATION TO THE ANALYSIS OF THE COMPACTION OF A POROUS BODY

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UDC 539.3

This paper considers a model of a plastically compressible porous medium with a cylindrical-type yield condition and its associated constitutive relations, which ensure independent mechanisms of shear and compaction of the porous material. This allows one to use the well-known theorems of plastic theory to analyze plastically compressible media and obtain analytical solutions for a number of boundary-value problems, including those taking into account conditions on strong-discontinuity surfaces. Results from full-scale studies of the structural periodicity of noncompact materials using wavelet analysis were employed to choose a physical model for a porous body and determine the properties and dimensions of a representative volume. The problem of extrusion of a porous material through a conical matrix was solved.

In plastic theory for porous bodies, two basic approaches are currently being developed: the phenomenological approach [1], based on experimental studies of flow curves for porous bodies, whose structural features are not considered, and the structurally phenomenological approach [2], in which a physical model of a deformable body is constructed taking into account structural features, and the constitutive relations are then verified experimentally. In this case, the deformation of porous bodies can be considered within the framework of the mechanics of structurally inhomogeneous media. At the mesolevel, the plastic-flow carriers are grains (conglomerates of grains) and pores, which, in aggregate, constitute a representative mesovolume [3]. In the present paper, we consider constitutive relations for a representative cell (mesovolume) of a plastically compressible body. The continual model of the mechanics of continuous media was used to describe the deformation of a macrovolume composed of spatially homogeneous microvolumes with different properties.

In the model of a structurally inhomogeneous body, the problem of choosing a representative volume is solved, generally speaking, ambiguously. There are various approaches to this problem: from averaging of the macrocharacteristics of the entire examined body over the representative volume to the consideration of the stochastic characteristics at the macro- and microlevels using the theory of random functions [2]. In the development and identification of the model, it is required to reveal the regularities of the structure of the material, including the microlevel, to determine the spatial repetition frequency of elements (grains, pores, and defects), statistical characterization of their distribution and anisotropy parameters, detection of the presence of scale invariance, etc. In this case, it is structural periodicity that determines the choice of a representative volume. The periodicity in real structures of porous materials was established experimentally in [4] using wavelet analysis.

1. Yield Conditions for Porous Bodies. In solving boundary-value problems of the mechanics of pressure treatment of noncompact materials, it is necessary to allow for irreversible volumetric compressive (tensile) strains. The limiting-state pyramid for a soil model [5] was considered even by Coulomb. The yield condition for a plastically compressible medium in the form of a Coulomb pyramid bounded by a plane of constant hydrostatic pressure (Coulomb–Mohr pyramid) was used in [1, 6]. Mises and Schleiher proposed an yield condition that in the stress space corresponds to a circular cone bounded by a plane of constant hydrostatic pressure (Mises–Schleiher cone) [5]. The indicated yield conditions are piecewise smooth. Ivlev and Bykovtsev [1] showed that the constitutive

Institute of Mechanical Engineering, Ekaterinburg 620219. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 43, No. 3, pp. 140–151, May–June, 2002. Original article submitted July 3, 2001; revision submitted January 28, 2002.

relations for such models can be considered using the theory of a strengthened plastic body with singular loading surfaces.

A smooth yield condition for porous bodies is proposed in [7]. It is assumed that the plastic compressibility of a deformed material is due to variation in the total volume of cracks and pores. This statement and the assumption of statistical homogeneity of material were used to introduce a loading surface of elliptic type for a porous body:

$$\alpha_1\tau^2 + \alpha_2\sigma^2 - \sigma_s^2 = 0, \qquad \alpha_1 = \alpha_1(\theta), \qquad \alpha_2 = \alpha_2(\theta).$$

Here $\tau = \sqrt{0.5 s_{ij} s_{ij}}$ is the shearing-stress intensity ($s_{ij} = \sigma_{ij} - \sigma\delta_{ij}$, where σ_{ij} are the stress-tensor components), $\sigma = \sigma_{ii}/3$ is the mean normal stress, σ_s is the yield stress for uniaxial tension, and $\theta = 1 - \rho$ is the porosity (ρ is the relative density).

Smooth yield conditions are proposed in [8–10] using similar assumptions on the nature of plastic deformation of a material with cylindrical and spherical pores. In the absence of porosity, the indicated smooth yield conditions coincide with the Mises yield condition for compact materials.

Druyanov [6] proposed an yield condition of cylindrical type:

$$\tau^2 + \sigma^2 = \beta^2\tau_s^2, \qquad \beta = \beta(\theta), \qquad \tau_s = \sigma_s/\sqrt{3}.$$

This condition is simpler than the yield condition of the elliptic type.

The most general case of deformation for the given condition occurs at the edge of the yield surface, which is the line of intersection of the Mises cylinder (state of pure shear) and the "bottom" (state of volumetric tension or compression).

The cylindrical yield condition differs from the elliptic yield condition primarily in the absence of a dilatation relation linking the characteristics of volumetric and shear strains. The corresponding model of a compacted body is a model with independent shear and compaction mechanisms, which simplifies the definition of the compressive and shear yield stresses in the form $\alpha_2 = \alpha_2(\theta)$ and $\alpha_1 = \alpha_1(\theta)$, respectively. The indicated moduli can be determined separately using independent models [theoretical and (or) full-scale] for the stress state because they are not linked by a dilatation relation. The cylindrical yield condition is appropriate for solving boundary-value problems of the mechanics of plastic flow of porous media in the cases where dilatation is absent or negligible. For example, the absence of compaction in the die hole during extrusion of a porous material was pointed out in [6]. In addition, the use of the cylindrical yield condition simplifies the solution of boundary-value problems with discontinuous strain and stress fields.

2. Physical Model of a Deformable Porous Body. The physical model of a plastically compressible body studied in the present paper is treated as a deterministic system in the context of the mechanics of structurally inhomogeneous (heterogeneous) media [11]. The validity of this approach is confirmed by studies of the structure of surfaces of briquettes produced by compaction of a titanium sponge using wavelet analysis, and the detected periodicity of real, outwardly random structures of materials at the microlevel with spatial repetition frequency of elements of about 3–5 grain diameters [4].

For the body considered, the following assumptions are adopted:

— The pore sizes are many times the molecular-kinetic dimensions of the crystal lattice of the skeleton and are many times smaller than the distances at which the macrocharacteristics of the medium change significantly;

— The mixture is monodisperse, the pores are present at each elementary volume in the form inclusions of a certain average size;

— The effects related to the pulsation, rotation, and translation of pores are absent as well as mass transport from the gas phase to the solid phase (skeleton) and back.

The packing model for a medium composed of particles of different sorts is shown in Fig. 1. The average porosity in the selected elementary volume is defined by the expression $\tilde{\theta} = \dfrac{1}{N}\sum\limits_{\theta_1}^{\theta_{\max}} \theta_i \tilde{n}(\theta)$, where k_i is the characteristic pore size (Fig. 2).

Using the indicated assumptions and the structurally phenomenological approach, Zalazinskii [12] considered a model of a medium which represents a conglomerate of statistically homogeneous, close-packed particles of isometric shape with discontinuity flaws localized on its boundaries. After deformation, the particles take the shape of polyhedra and form regular structures — lattices similar to crystallographic ones [13]. Defects — pores filled with gas — are located at the lattice points. A diagram of the packing is given in Fig. 3a.

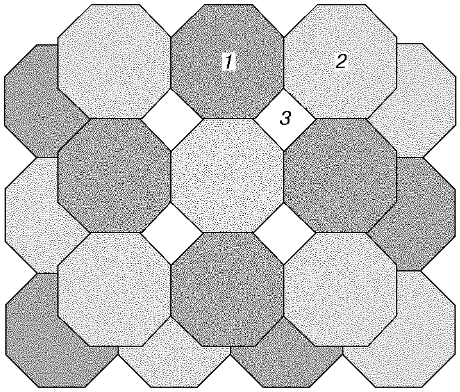

Fig. 1

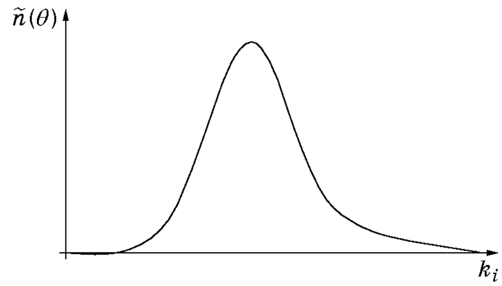

Fig. 2

Fig. 1. Packing model for particles of different sorts: 1 and 2 are structure Nos. 1 and 2, respectively, and 3 is the pore.

Fig. 2. Pore size distribution.

During plastic compaction of close-packed particles which initially had point contacts to one another, the particles fill in the free space (pores). Under loading, the point contacts become contact surfaces; the shape of the particles changes, becoming similar to the shape of polyhedra; the pores decrease, remaining polyhedra to a certain moment, and then, at small porosity, they assume a spherical shape [12].

In the examined model of a porous compacted body, it is necessary to determine the developed plastic flow loads and the changes in the shape and volume of the pores. By virtue of the statistical homogeneity of properties of the deformable material, in order to derive physical equations, it suffices to solve the problem for a characteristic cell that represents a statistical ensemble.

For definiteness, in orthogonal coordinates, we consider a certain volume filled with macroparticles in the shape of polyhedra (Fig. 3b). Each cell of a plastically compressible media exhibits piecewise-homogeneous properties and consists of tetrahedra, which form a rigid-plastic skeleton and occupy volume $\Omega = \omega_f \cup \omega_p$ (ω_f is the volume of a particle and ω_p is the volume of a pore). The outer surface of the cell $S = S_F \cup S_V$ is loaded by a system of surface forces $\lambda F = \{\lambda F_i\}$ (λ is an uncertain factor, which increases from zero). The boundary conditions are written as $\sigma_{ij} n_j = \lambda F_i$ ($x \in S_F$) and $v_{is} = v_{is}^0$ ($x \in S_V$). For the cell inside the volume Ω, the relationship between the components of the stress deviator and the strain rate tensor is given by

$$s_{ij} = 2\tau_s(x) e_{ij}/H(x), \tag{1}$$

where $\tau_s(x) = \tau_s$ for $x \in \omega_f$ and $\tau_s(x) = 0$ for $x \in \omega_p$, $e_{ij} = \xi_{ij} - \xi\delta_{ij}/3$ (ξ_{ij} are the strain rate tensor components), and $H = \sqrt{2e_{ij}e_{ij}}$ is the strain-rate intensity.

With increase in external forces, the cell enters the general yield state when the load reaches the limiting value $\lambda^* F$. To determine ultimate load, it is generally necessary to use the ideal plastic limit theorem for structurally inhomogeneous bodies [12, 14]:

$$\int\limits_S F_i^0 v_i \, ds \leqslant \sum_{m=1}^{M} \int\limits_{\omega_m} [\tau_{sn}(k_i,\theta) H + \sigma_n(k_i,\theta)\xi]_i \, d\omega. \tag{2}$$

Here the varied quantities are H and ξ, M is the number of cells, and m is the cell number.

Inequality (2) contains a varied function of the strain field. Upon transition from the kinematically permissible state of the cell to a real state, the inequality becomes the equality.

3. Derivation of Constitutive Relations. In the derivation of constitutive relations, we calculated energy dissipation for the tetrahedra included in the unit cell, using relations of the finite element method. It was assumed that all tetrahedra, except for the tetrahedron representing a pore, were rigid bodies and moved relative to one another by sliding along their faces which are acted upon by normal and shearing stresses. The shearing stress value was set equal to the shear yield stress of the compact material, and the normal stresses were determined from

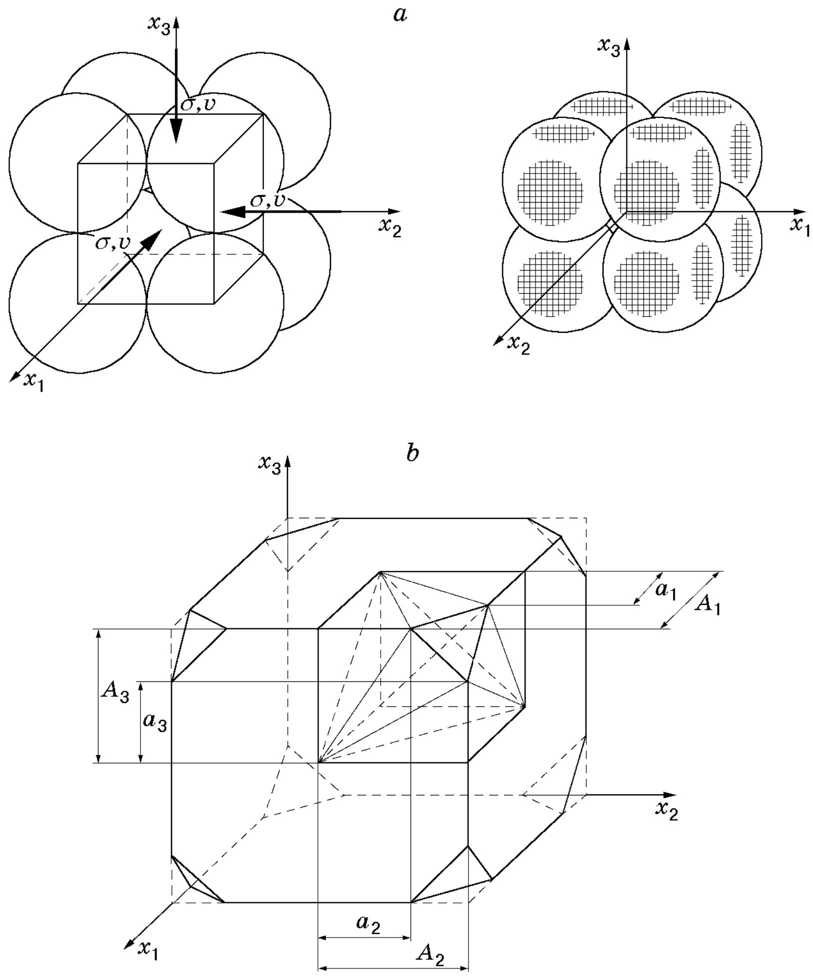

Fig. 3. Diagram of particle packing in the model of a porous body (a) and finite-element discretization of the cell (b).

the equilibrium equations for the tetrahedra. The calculations were carried out for the case of volumetric uniform compression of the porous cell in the field of mean stress σ and for the case of strain of pure shear along the planes weakened by defects (pores) of polyhedral or spherical shapes.

In the calculation of the gas pressure in a pore q, it was assumed that removal of the gas from the pores was ensured by filtration until attainment of a certain critical porosity θ^*, after which the gas pressure increased according to the equation

$$q = q^*(\theta^*/\theta)(1-\theta)/(1-\theta^*),$$

where θ^* and q^* are the porosity and internal pressure, respectively, in the pores at the moment the removal of the gas ceases ($\theta < \theta^*$).

Calculation and approximation of results yielded the following relations for the compressive σ_s^* and shear yield stresses τ_s^*:

$$\sigma_s^* = -(2/\sqrt{3})\tau_s \ln(\eta/\theta) + q, \qquad \tau_s^* = \tau_s(1 - K\theta^{\zeta}) \tag{3}$$

(K, η, and ζ are parameters that characterize the geometry of the defects). The calculations showed that the indicated parameters can take the following values: $1 \leqslant K \leqslant 1.63$, $\eta \leqslant 1$, and $\zeta = 2/3$. For randomly located defects of isometric shape, $K = 1$, $\eta = 1$, and $\zeta = 2/3$.

Relations (3), obtained by solving test problems of determining the compressive and shear yield stresses for a porous body, are general in nature. Real deformation processes, taking into account plastic compressibility, are characterized not only by a decrease of porosity but also by a change in the shape of particles and pores. At the initial moment, a porous body can be treated as an isotropic medium with macrocharacteristics averaged over the volume. Under deformation, irrespective of the scheme (except for the trivial case of volumetric compression), porous bodies inevitably acquire a certain degree of anisotropy. Neglect of the latter can introduce large errors into results of mathematical modeling. The parameters K, η, and ζ depend not only on the porosity and shape of the pores but also on the nature of anisotropy of the medium. Saltykov [15] showed that the properties of the materials characterized by three-dimensional packing of particles are determined with reasonable accuracy by the structural parameters of the principal planes of the material deformed. Generally, to describe the regularities of plastic deformation for porous materials, it is necessary to specify the structural parameters of the characteristic volume element for the three principal planes. In the principal axes, Eq. (1) becomes

$$\sigma_{11} = -(2/\sqrt{3})\tau_s \ln(\eta_{11}/\theta) + 2\tau_s(1 - K_{11}\theta^{\zeta_{11}})(\xi_{11} - \xi/3)/H,$$

$$\sigma_{22} = -(2/\sqrt{3})\tau_s \ln(\eta_{22}/\theta) + 2\tau_s(1 - K_{22}\theta^{\zeta_{22}})(\xi_{22} - \xi/3)/H,$$

$$\sigma_{33} = -(2/\sqrt{3})\tau_s \ln(\eta_{33}/\theta) + 2\tau_s(1 - K_{33}\theta^{\zeta_{33}})(\xi_{33} - \xi/3)/H.$$

In this case, in formulas (3), we have $K = \prod_{i=1}^{3} K_{ii}$, $\zeta = \prod_{i=1}^{3} \zeta_{ii}$, and $\eta = \prod_{i=1}^{3} \eta_{ii}$.

The above relations and the cylindrical yield condition were used to solve a number of problems, in particular, the problems of two-side compaction of a porous body and extrusion of a porous material in an axisymmetric formulation [12].

4. Strong-Discontinuity Surfaces in a Plastically Compressible Medium. The theory of discontinuous solutions was originally developed for problems of hydrodynamics. For discontinuities of the type of shock waves, exact analytical dependences, known as the Rankine–Hugoniot relations, were obtained. The indicated relations were used to solve plastic problems with the Mises yield condition [5] and were considered in many other papers. The conditions on surfaces of strong discontinuity of rates for plastically compressible media were studied in [6, 16, 17].

Following [18], we consider a discontinuous solution as a sequence of continuous motions for the system of equations of a plastically compressible medium. In thin layers with continuous but abrupt variation of motion characteristics, we specify appropriate external actions, heat influxes, and other types of energy. In passing to the limits, we assume that the total characteristics of additional external actions in the form of mass forces can have, generally speaking, nonzero values. In this case, the basic equations of the mechanics of continuous media — the continuity, momentum, and energy equations — should hold. The momentum equation are eliminated from consideration because for momenta that are external for the medium, the surface distribution density is always equal to zero.

In constructing discontinuous solutions in plastic theory, it is commonly assumed that mass forces are absent. However, as is indicated in [18], discontinuous solutions can be constructed by introducing appropriate external actions that are consistent with the adopted model of the medium. Although a few types of special mass forces are encountered in practice, it is impossible to characterize the class of all possible fields of these forces. Generally, the magnitude of these forces is not subjected to any restrictions and is determined from the momentum equation [19]. In constructing discontinuous solutions for a multicomponent medium, mass-force components are introduced, in particular, in [20]. In this case, it is pointed out that the nature of these forces must be specified for the physical process studied. For models of two-component media "gas–solid particles," similar actions in the form of surface-force components at a discontinuity (and the additional term in the energy equation associated with the indicated force) are considered in [21] and [9, 22]. In this case, the magnitude of the surface force was chosen from the condition of specified flow regimes at discontinuities. Thus, for the passage of a multicomponent medium through a discontinuity surface, the constitutive relation is the relation between the interphase interaction forces and the solid-phase particle interaction forces. Therefore, the conditions on a strong-discontinuity surface in a plastically compressible medium can be considered in terms of energy, without assuming that the mass forces are equal to zero.

Let us consider a discontinuity surface S_h, at whose point M, the velocity along the normal to the surface is equal to $v_s = \lim\limits_{\Delta t \to 0} \Delta s/\Delta t$ (Δs is the distance traveled by the point M along the normal for time Δt) (Fig. 4). At the point M, we introduce a moving local coordinate system (n, τ, z), which moves uniformly, rectilinearly, and translationally. At the time considered, it has velocity equal to the velocity of the point M. Let the material flow

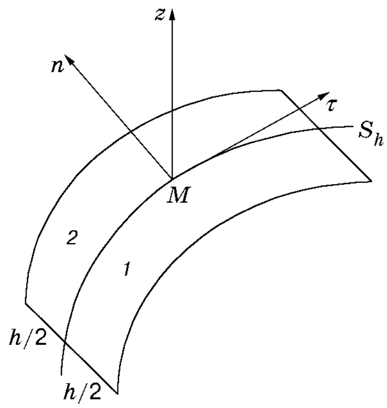

Fig. 4. Discontinuity surface.

from region 1 of the discontinuity surface to region 2 and the n axis be directed normally to S_h toward region 2. Then, in the fixed coordinate system, the general integral equations of continuity, momentum, and energy lead to the following relations (subscripts 1 and 2 correspond to the regions of the discontinuity surface S_h):

$$\rho_1(v_s - v_{n1}) = \rho_2(v_s - v_{n2}); \tag{4}$$

$$\boldsymbol{p}_{n2} - \boldsymbol{p}_{n1} = \rho_1 \boldsymbol{v}_1(v_s - v_{n1}) - \rho_2 \boldsymbol{v}_2(v_s - v_{n2}) - \boldsymbol{R}; \tag{5}$$

$$W + \boldsymbol{p}_{n2} \cdot \boldsymbol{v}_2 - \boldsymbol{p}_{n1} \cdot \boldsymbol{v}_1 = \rho_1(v_s - v_{n1})(v_1^2/2 + U_1) - \rho_2(v_s - v_{n2})(v_2^2/2 + U_2) + q_{n2}^* - q_{n1}^*. \tag{6}$$

Here $\boldsymbol{p}_n$ is the stress vector, U is the internal energy density, $\lim\limits_{h\to 0} \int\limits_V \rho \boldsymbol{F}\, d\tau = \int\limits_{S_h} \boldsymbol{R}\, d\sigma$, $\boldsymbol{R}$ is the surface distribution density of external mass forces on S_h, $\lim\limits_{h\to 0} \int\limits_V \Big(\rho \boldsymbol{F} \cdot \boldsymbol{v} + \rho \dfrac{dq_{\rm m}^*}{dt}\Big)\, d\tau = \int\limits_{S_h} W\, d\sigma$, W is the power flux density of mass forces $\rho\boldsymbol{F}$, q_n^* is the total external influx of additional specific energy, and $dq_{\rm m}^*/dt$ is the total specific additional energy influx due to mass sources per unit time.

Converting to vector components, we write conditions (5) and (6) as

$$[p_{ni}] = -\rho(v_s - v_n)[v_i] - R_i, \qquad W + \sum_i [p_{ni} v_i] - [q_n] = -\frac{\rho(v_s - v_n)}{2}\Big(\sum_i [v_i^2] + [U]\Big), \qquad i = (n, y, z),$$

where square brackets denote a jump of a quantity and $[v] = v_2 - v_1$.

We specify the heat flux by the Fourier law $q_i = -\alpha \operatorname{grad} T$ (α is the thermal conductivity) and designate $W' = W - \dfrac{\rho(v_n - v_s)}{2} \sum\limits_i [v_i^2]$. After transformations, we obtain the expression

$$W' + \sum_i [p_{ni} v_i] = \sigma_2 \xi + \tau_2 H - \lim_{h\to 0} \int\limits_V \Big(\frac{\partial \sigma_{nn}}{\partial n}(v_{n1} - v_n) + 2\frac{\partial \sigma_{ny}}{\partial n}(v_{y1} - v_y) + 2\frac{\partial \sigma_{nz}}{\partial n}(v_{z1} - v_z)\Big)\, d\omega = \sigma_2 \xi + \tau_2 H - I,$$

where

$$I = \lim_{h\to 0} \Bigg\{ \int\limits_V \Big\{\frac{\partial \sigma}{\partial n}(v_{n1} - v_n)\Big\}\, d\omega + \int\limits_V \frac{\partial \tau}{\partial n}\Big\{\frac{e_{nn}(v_{n1} - v_n) + 2e_{ny}(v_{y1} - v_y) + 2e_{nz}(v_{z1} - v_z)}{H}\Big\}\, d\omega$$

$$+ \sum_i \frac{a_i}{6} \int\limits_V \frac{\tau}{H^3}\Big\{\Big(\frac{\partial^2 v_i}{\partial n^2}\frac{\partial v_j}{\partial n} - \frac{\partial^2 v_j}{\partial n^2}\frac{\partial v_i}{\partial n}\Big)\Big\}\Big\{\Big(\frac{\partial v_j}{\partial n}(v_{i1} - v_i) - \frac{\partial v_i}{\partial n}(v_{j1} - v_j)\Big)\Big\}\, d\omega\Bigg\}, \tag{7}$$

$$\begin{pmatrix} i \\ j \end{pmatrix} = \begin{pmatrix} n\ y\ z \\ y\ z\ n \end{pmatrix}, \qquad a_n = a_z = 4, \quad a_y = 3.$$

Let us consider the integrals under the summation sign on the right side of (7). In the neighborhood of the point M, the velocity components v_i are expanded in a Taylor series in the coordinate n:

$$v_i = v_{i1} + \sum_{m=1}^{M} \frac{\partial^m v_i(0)}{\partial n^m}\,\frac{n^m}{m!}.$$

We assume that for each pair of subscripts (i, j), the following relation holds: $\partial^m v_i(0)/\partial n^m = \beta_{ij}\,\partial^m v_j(0)/\partial n^m$ ($\beta_{ij} = \text{const}$). Then, the expressions in braces are equal to zero for all three integrals, and the last term on the right side of (7) can be neglected.

Let us consider the first integral on the right side of (7). Expanding σ and v_n in Taylor series and setting $\partial^m\sigma(0)/\partial n^m = \beta_{\sigma n}\,\partial^m v_n(0)/\partial n^m$ ($\beta_{\sigma n} = \text{const}$), we obtain

$$\lim_{h\to 0}\int\limits_V \Big\{\frac{\partial\sigma}{\partial n}\,(v_{n1} - v_n)\Big\}\,d\omega = -\frac{1}{2}\,(\sigma_2 - \sigma_1)(v_{n2} - v_{n1}) = -\frac{1}{2}\,[\sigma]\xi.$$

Similarly, for the second integral on the right side of (7), we have

$$\lim_{h\to 0}\int\limits_V \frac{\partial\tau}{\partial n}\Big\{\frac{e_{nn}(v_{n1} - v_n) + 2e_{ny}(v_{y1} - v_y) + 2e_{nz}(v_{z1} - v_z)}{H}\Big\}\,d\omega = -\frac{1}{2}\,[\tau]H.$$

As a result, the power dissipation rate D_h on the discontinuity surface is equal to

$$D_h = \frac{1}{2}\,((\sigma_2 + \sigma_1)\xi + (\tau_2 + \tau_1)H) - [q_n] + \frac{1}{2}\,\rho(v_n - v_s)\sum_i [v_i^2]. \qquad (8)$$

In solving boundary-value problems, it is more reasonable to express the quantity D_h in terms of the yield stresses σ_s^* and τ_s^*. When the cylindrical yield condition is used, the passage to the limit is obvious because the conditions $\sigma = \sigma_s^*$ and $\tau = \tau_s^*$ should be satisfied. For the elliptic yield condition, we use the following formulas [6]:

$$\sigma = (\xi/Q)(\sigma_s^*)^2, \quad \tau = (H/Q)(\tau_s^*)^2, \quad \bar{D} = Q = (H^2(\tau_s^*)^2 + \xi^2(\sigma_s^*)^2)^{1/2}.$$

For the power dissipation rate, the elliptic yield condition gives a more complex expression than (8):

$$\bar{D}_h = (H^2(\tau_{s2}^*)^2 + \xi^2(\sigma_{s2}^*)^2)^{1/2} - \frac{1}{2}\,\{H^2((\tau_{s2}^*)^2 - (\tau_{s1}^*)^2) + \xi^2((\sigma_{s2}^*)^2 - (\sigma_{s1}^*)^2)\}^{1/2} - [q_n] + \frac{1}{2}\,\rho(v_n - v_s)\sum_i [v_i^2]. \qquad (9)$$

Relations (8) and (9) are used in solving problems of developed plastic flow, in which the elastic components of stress tensors and strain rates can be ignored. For the cases where it is necessary to allow for elastic stresses and strains, similar results are obtained in [16, 17]. Burenin et al. [16] studied a yield condition in the form of a Coulomb–Mohr pyramid. Sadovskii [17] examined the properties of the Prandtl–Reiss equations for dynamic problems.

5. Extrusion of a Porous Material. We construct a solution of the boundary-value problem of extrusion of a porous material through a conical die (Fig. 5) under plane deformation with a yield condition of cylindrical type. Friction is ignored. We assume that plastic strains are concentrated on the discontinuity lines OA and OB, which are interfaces between regions 1, 2, and 3 moving as rigid bodies. Let the relative densities and velocities in each zone be equal to ρ_i and v_{ji} ($j = x, y$; $i = 1, 2, 3$), respectively. We assume that ρ_1 and v_1 are specified. Then, the simplest kinematically permissible velocity field is written as (see, e.g., [23])

$$v_{x1} = v_1, \quad v_{x2} = v_2\cos\gamma, \quad v_{x3} = v_3, \quad v_{y1} = 0, \quad v_{y2} = -v_2\sin\gamma, \quad v_{y3} = 0.$$

Converting to the local axes (n, τ) on the discontinuity lines, for jumps of the velocity components, we obtain the expressions

$$\begin{aligned}
[v_n] &= (v_2\cos\gamma - v_1)\sin\varphi - v_2\sin\gamma\cos\varphi, \\
[v_\tau] &= -(v_2\cos\gamma - v_1)\cos\varphi - v_2\sin\gamma\sin\varphi \quad \text{on OA}, \\
[v_n] &= (v_3 - v_2\cos\gamma)\sin\psi - v_2\sin\gamma\cos\psi, \\
[v_\tau] &= (v_3 - v_2\cos\gamma)\cos\psi + v_2\sin\gamma\sin\psi \quad \text{on OB}.
\end{aligned} \qquad (10)$$

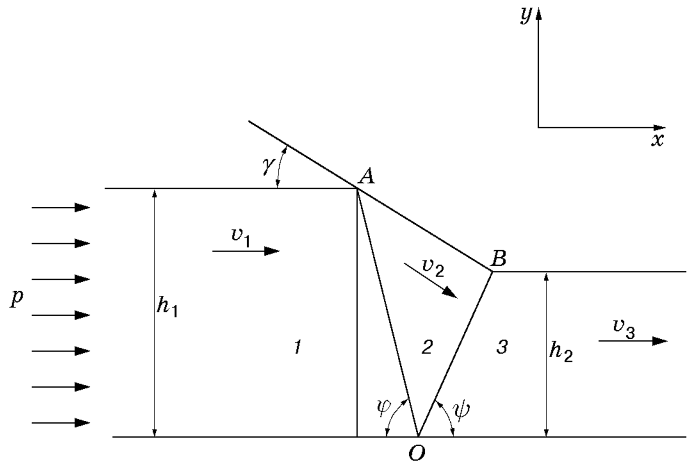

Fig. 5. Diagram of the calculation of extrusion of a porous material.

The velocities v_2 and v_3 are determined from the kinematic compatibility conditions [23]: $[v_x]-[v_y]\tan\varphi = a$ on OA and $[v_x]+[v_y]\tan\psi = b$ on OB. In this case, at the point O, the following conditions should be satisfied: $[v_x] = v_3 - v_1$ and $[v_y] = 0$. Hence it follows that

$$v_2 = \frac{1}{\sin\gamma}\,\frac{v_1}{\cot\gamma - \tan\psi}, \qquad v_3 = v_1\,\frac{\cot\gamma + \tan\varphi}{\cot\gamma - \tan\psi}.$$

Formulas (10) can be written as

$$[v_n]_1 = v_1\sin\varphi\,\frac{\tan\psi - \cot\varphi}{\cot\gamma - \tan\psi}, \quad [v_\tau]_2 = -v_1\sin\varphi\,\frac{1+\cot\varphi\tan\psi}{\cot\gamma - \tan\psi} \quad \text{on OA},$$

$$[v_n]_1 = v_1\sin\psi\frac{\tan\varphi - \cot\psi}{\cot\gamma - \tan\psi}, \quad [v_\tau]_2 = v_1\sin\psi\,\frac{1+\cot\psi\tan\varphi}{\cot\gamma - \tan\psi} \quad \text{on OB}.$$

The densities ρ_2 and ρ_3 are determined directly from the condition $\rho_i v_{ni} = \text{const}$ but it is easier to use the formulas

$$\rho_1 v_{n1} = \rho_2([v_n]_1 + v_{n1}), \qquad v_{n1} = v_1\sin\varphi, \qquad \mu\rho_1 v_1 = \rho_3 v_3,$$

where $\mu = h_1/h_2$ is the reduction (extract).

To eliminate the angle φ from the equations, we use the kinematic relation $(h_1 - h_2)\cot\gamma = h_1\cot\varphi + h_2\cot\psi$ or $\cot\varphi = \cot\gamma - (\cot\gamma + \cot\psi)/\mu$.

With allowance for (8), the equation for calculating the extrusion force is written as

$$p/\tau_s = \{(\sigma_1+\sigma_2)(\tan\psi - \cot\varphi) + (\tau_1+\tau_2)[4(\tan\psi-\cot\varphi)^2/3 + 1 + \cot\varphi\tan\psi]^{1/2}$$

$$+ [(\sigma_2+\sigma_3)(\tan\varphi - \cot\psi) + (\tau_2+\tau_3)[4(\tan\varphi-\cot\psi)^2/3 + 1 + \cot\psi\tan\varphi]^{1/2}]/\mu\}/(2(\cot\gamma - \tan\psi)), \qquad (11)$$

where the dependences $\sigma_i = \sigma_i(\theta_i)$ and$\tau_i = \tau_i(\theta_i)$ are determined according to (3) ($\sigma = \sigma_s^*$, $\tau = \tau_s^*$, and $\theta = 1-\rho$).

For an incompressible material, it suffices to set $[v_n]_i = 0$, eliminating the terms due to irreversible volume changes from formula (11).

The calculations are carried out for two intervals of initial porosity: $\theta_1 = 0.2$–0.4 (moderate porosity) and 0.06–0.12 (low porosity). The cone angle γ is set equal to 45°. The value of the angle is chosen for the following reasons. As is known, the best power parameters of the extrusion process are ensured for $30° < \gamma < 45°$ (see, e.g., [6, 12]). At the same time, for small angles γ and large μ, the accuracy of pressure determination by the adopted calculation scheme reduces considerably. In this case, it is recommended to complicate the calculation scheme by increasing the number of rigid blocks [24]. From the calculations it follows that for $\gamma = 30°$ and $\mu \geqslant 10$, the pressure increases sharply.

Calculation results are shown in Figs. 6 and 7. The quantity μ was varied in the range of 6–10. In the calculations, it was assumed that $K = 1$, $\eta = 1$, and $\zeta = 2/3$.

From an analysis of the results it follows that at moderate initial porosity, extrusion without compaction is possible for minimum reduction (see Fig. 6c). In this case, the pressure corresponds to an incompressible material,

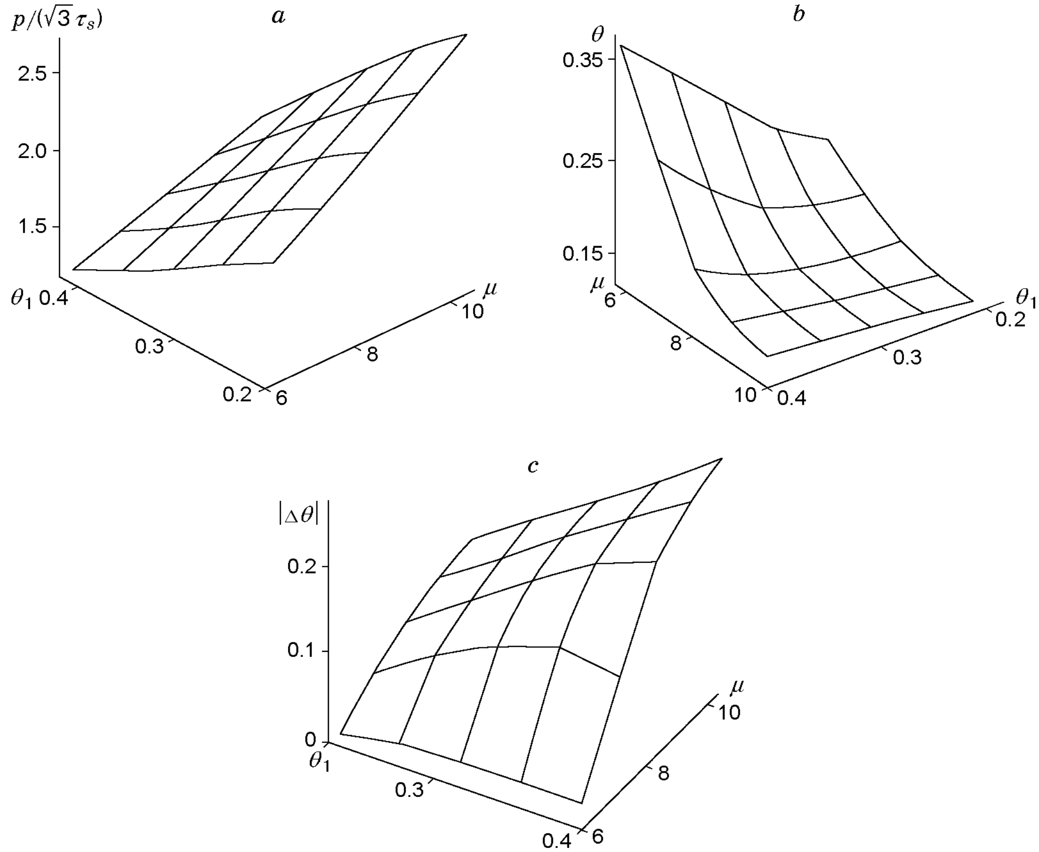

Fig. 6. Calculated extrusion parameters at an initial porosity $\theta_1 = 0.2$–0.4: (a) pressure $p/(\sqrt{3}\tau_s)$; (b) residual porosity; (c) jump of porosity.

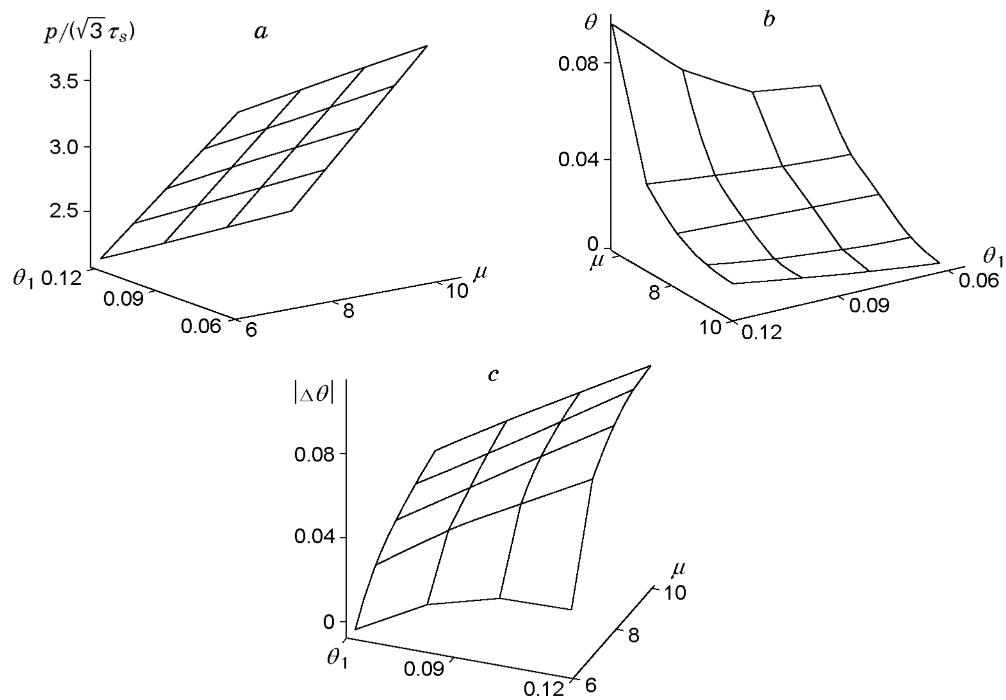

Fig. 7. Calculated extrusion parameters at an initial porosity $\theta_1 = 0.08$–0.12: (a) pressure $p/(\sqrt{3}\tau_s)$; (b) residual porosity; (c) jump of porosity.

for which it is necessary to set $\tau_s = \tau_s^*(\theta_1)$. With increase in μ, the porosity decreases sharply: $\theta = 0.12$–0.15 (see Fig. 6b). A similar pattern is observed for low initial porosity, although its jump in absolute value is much smaller than that in the first case. For reduction $\mu = 10$, a nearly nonporous state ($\theta < 0.01$) (see Fig. 7b) is reached. In this case, $p/(\sqrt{3}\tau_s) = 3.7$–4.0, which is in fair agreement with calculated and experimental data for a wire produced from a titanium sponge [12].

This work was supported by the Russian Foundation of Fundamental Research (Grant No. 01-01-96465) and the Administration of the Sverdlovsk Region.

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